28- Thermoelectric modeling of a cooling module with heat exchangers. J. G. Stockholm, D.W. Stockholm. in proceedings of XI th International Conference on Thermoelectrics, (The University of Texas at Arlington, Arlington Texas 76019 , Dept of Electrical Engineering Editor K.R. Rao P.O. Box 19016: p 140-146) Arlington, Texas, USA 7-9 October 1992.

THERMOELECTRIC MODELING OF A COOLING MODULE WITH HEAT EXCHANGERS

John G. Stockholm. Daniel W. Stockholm.

Marvel S.A. 11 rue J. du Bellay, 78540 Vernouillet, France

ABSTRACT

Module manufacturers' brochures give the performances of their modules as a function of the temperatures of the cold and hot sides of the modules and as a function of the electrical current through the module.

The user is concerned with the temperatures in his equipment. He or she knows on the cold side, either a temperature of a surface, or of a fluid and on the hot side generally the temperature of a fluid.

A simple thermal thermoelectric model of a cooling module with a heat exchanger on each side is presented.

The object of this model is to give people who use thermoelectric modules an easy way to know the performances of a module with a heat sink (heat exchanger) that has a known thermal resistance.

An easy way to solve the model with a spreadsheet is presented with two examples: a thermoelectric module on its own and one with a heat sink. The outputs are the cooling, heating and electrical powers, the temperatures at the interfaces of the module are given so that the user can compare the values with those from a module manufacturer.

The symbol * is used as the multiplication sign. The word thermoelectric is abbreviated by TE.

1. THERMAL THERMOELECTRIC MODEL

A schematic of the TE module thermal model is shown in Fig. 1

The thermal thermoelectric mathematical model consists of a thermoelectric module with a heat exchanger on each side. The energy balance consists of 6 equations with 6 unknowns. The equations are presented in paragraph 5 Equations.

The fundamentals of thermoelectric equations are given in ref: 1 We have chosen the signs so that the cooling power and the COP are negative and the heating and electrical powers are positive.

We have chosen for clarity a most simple model, which is sufficient for most applications.

The assumptions for the mathematical model are :

- Each ceramic layer has the same temperature as the faces of T.E elements in contact with them
- All the T.E elements are electrically connected in series and thermally connected in parallel.
- Joule power (Re*i^2) produced in the T.E elements is dissipated equally at the ends of the two T.E elements (Re*i^2)/2. This is correct when the TE material properties are a linear function of temperature, the temperature dependence of the material used has a small term of the second order which is negligible over the ΔT of the TE element.
- The only heat losses outside of the module are expressed as an exterior thermal conductance C_xt, between the cooled base temperature and the heated base temperature. They are represented in Fig. 1 as a seal, which includes a tightening mechanism, such as screws.

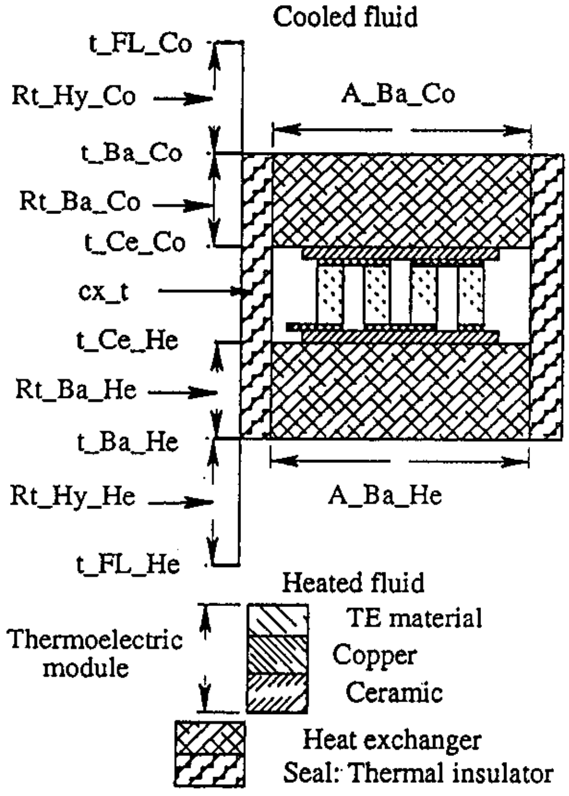


Fig. 1 Schematic of TE module - Thermal model

Practical experience has shown that the conduction losses C_xt between the heated side and the cooled side which are outside of the thermoelectric material are well approximated by using the temperature t_Ba of the bases. The temperature of the base for a liquid heat exchanger is the temperature of the wall constituting the passage for the liquid. For a gas it is the average temperature of the base of the fins, no fins are shown in Fig. 1.

The characteristics of a thermoelectric module and of the heat exchangers are examined.

2. NOTATIONS

The notations used for the variables or parameters are as mnemonic as possible. They are composed of 2 or 3 parts (basic notations) each part is separated by a " _ " .

t_Ba_Co: t stands for temperature, **Ba** stands for base and **Co** stands for cooled

Basic notations: Meaning of the different terms

Symbol	units	Designation
A	m^2	Area
Ba	m^2	Base of the heat exchanger
C	W/K	Thermal Conductance
Ce		Ceramic
Co		Cooled side
COP	non dim.	Coefficient Of Performance
eL		eLectrical
Fin_eff	non dim.	Fin efficiency
FL		FLuid
GF	m	Geometric Factor of TE element
He		Heated side
h	W/(m2*K)	convection coefficient **h**
i	A	Electrical current (intensity)
k	W/(m*K)	Thermal conductivity **k**
Mo		Module
Nb	non dim.	Number
P	W	Power
Ro	Ω*m	Electrical resistivity Rho
Re	Ω	electrical Resistance
Rt	K/W	thermal Resistance
S	V/K	Seebeck voltage
t	°C	temperature
Te		TE element
tm	°C	Average temperature (mean)
U	V	Voltage across module
xt		exterior

Variables:

A_Te	m^2	Area of TE element
C_xt	W/K	External thermal Conductance
C_Mo	W/K	Thermal conductance C of TE Module
GF	non dim.	Geometric Factor of TE element = A_Te/L_Te
h_Ba_Co	$W/(m^2*K)$	convection coefficient **h** on Base of Cooled side
h_Ba_He	$W/(m^2*K)$	convection coefficient **h** on Base of Heated side
k_Te	W/(m*K)	Thermal conductivity **k** of TE material
L_Te	m	Length of TE material
P_Co	W	Cooling Power
P_eL	W	eLectrical Power
P_He	W	Heating Power
Ro_Te	Ω*m	Electrical resistivity Rho of TE material
Re_Mo	Ω	electrical Resistance of TE Module
S_Te	V/K	Seebeck coefficient of TE material
S_Mo	V/K	Seebeck coefficient of TE Module
t_FL_Co	°C	temperature of the Cooled FLuid
t_FL_He	°C	temperature of the Heated FLuid
t_Ba_Co	°C	temperature of the Cooled Base
t_Ba_He	°C	temperature of the Heated Base
t_Ce_Co	°C	temperature of the Cooled Ceramic
t_Ce_He	°C	temperature of the Heated Ceramic
tm_Te	°C	Average temperature of the TE element (mean)
U	V	voltage across the module

3.THERMOELECTRIC MODULE CHARACTERISATION

Thermoelectric material is characterised by 3 parameters

Ro_Te	Electrical resistivity Ω*m
S_Te	Seebeck coefficient V/K
k_Te	Thermal conductivity W/(m*K)

These parameters are a function of temperature, generally one uses a polynomial correlation with terms of the second order in temperature. They represent the average values of the n and the p type materials.
The functions used are those given by Melcor Inc. They are given here as a function of (tm -23) so that the first term is the value at room temperature and the second term is the slope of the curve at 23 °C.

Ro_Te (tm)= (10.8497 +0.0535*(tm - 23)
+ 62.8E - 6*(tm-23)^2) / 10^6
S_Te(tm)= (210.9019 +0.34426*(tm-23)
- 0.9904E -3*(tm-23)^2) /10^6
k_Te(tm) = 1.65901 -3.32E-3(tm-23) +41.3E-6(tm-23)^2

A TE module can be characterised by

Re_Mod = Total electrical resistance Ω
S_Mod = Total Seebeck V/K
C_Mod = Thermal conductance W/K

These parameters can be measured directly on a TE moderistics will include the thermal properties of the ceramic ınd the electrical connectors etc. which are inside the nodule. This assumption is equivalent to saying that the emperature of the ceramic is the same as the temperature ›f the end of the TE element

thermoelectric module is characterised besides the maerial characteristics by two other parameters:

Nb_Te = number of TE elements in the module
GF = the geometric factor of the TE elements
= A_Te/L_Te
One can write for a module

Re_Mo = Nb_Te* Ro_Te/GF
S_Mo = Nb_Te*S_Te
C_Mo = Nb_Te*GF*k_Te

4. HEAT EXCHANGER CHARACTERISATION

'e have chosen for the heat exchanger to use thermal :sistances for two reasons:
because when they are in series they must be added.
when one does not exist the value is zero.
he model requires knowing the thermal resistance of ɔth heat exchangers. We have divided this thermal resisınce into two parts:
here is also the heat conduction C_xt that goes between ıe heat exchangers but outside the module.

1 Contribution due to convection coefficient

Thermal hydraulic resistances K/W: Rt_Hy_Co and t_Hy_He
t_Hy = 1 / (h_Ba *A_Ba) in K/W where
_Ba area of the base of the heat exchanger on the fluid de m^2 and **h_Ba** convection coefficient as seen by the ıse: $W/(m^2*K)$

ɔr air heat exchangers with fins one can calculate **t_Hy** in the following way.
ɔe base has an area of **A_Ba**, the fins on the base have ı area of **A_fin** and a fin efficiency of **fin_eff**, the ›nvection coefficient of the fins is **h** and we can write:

h_Ba = A_fin*fin_eff*h /A_Ba

'e neglect the area of the base between the fins in calcuting the area in contact with the fluid.

For liquid heat exchangers
e convection coefficient of the fluid **h_Ba is** at the inrface between the fluid and the walls of the duct.

2 the thermal conduction through a solid

Thermal base resistance K/W:
_Ba_Co and Rt_Ba_He
ıese values can be calculated(e.g.: by finite differences by finite element analysis) or measured.
thermal resistances is independent of the boundary contions. As these two thermal resistances are a function of e convection coefficient at the level of the base, they are should include the thermal interface resistance between the base and the ceramic.
The calculations of the above thermal hydraulic resistances and thermal resistances are done prior to using the model.

5. EQUATIONS

5.1. Thermal power that is pumped out of cooled fluid by module

Seebeck power	- S_Mo * i * (t_Ce_Co + 273)
Joule power	+ (Re_Mo * i^^{2})/2
Conduction in T . E	+ C_Mo * (t_Ce_He - t_Ce_Co)
Conduction outside module	+ C_xt * (t_Ba_He -t_Ba_Co)

P_Co = - S_Mo * i * (t_Ce_Co + 273)+ (Re_Mo * i^^{2})/2+
C_Mo * (t_Ce_He - t_Ce_Co)+
C_xt * (t_Ba_He -t_Ba_Co)

5.2. Thermal power exiting module that is entering the heated fluid

Seebeck power	S_Mo * i * (t_Ce_He + 273)
Joule power	+(Re_Mo * i^^{2})/2
Conduction in T.E	- C_Mo * (t_Ce_He - t_Ce_Co)
Conduction outside module	+ C_xt * (t_Ba_He -t_Ba_Co)

P_He = S_Mo * i * (t_Ce_He + 273)+(Re_Mo * i^^{2})/2
- C_Mo * (t_Ce_He - t_Ce_Co)
+C_xt * (t_Ba_He -t_Ba_Co)

5.3 Evaluation of ceramic temperature in contact with Cooled Base

The thermal resistances of heat exchanger and of fluid interface = Rt_Ba_Co + Rt_Hy_Co
t_Ce_Co = t_FL_Co + P_Co * (Rt_Ba_Co + Rt_Hy_Co)

5.4. Evaluation of ceramic temperature in contact with heated Base.

The thermal resistance of heat exchanger and of fluid interface = Rt_Ba_He + Rt_Hy_He
t_Ce_He = t_FL_He + P_He * (Rt_Ba_He + Rt_Hy_He)

5.5. Evaluation of Base temperature at interface with cooled fluid

t_Ba_Co = t_FL_Co + P_Co * Rt_Hy_Co

5.6. Evaluation of Base temperature at interface with heated fluid

t_Ba_He = t_FL_He + P_He * Rt_Hy_He

6. INPUT VARIABLES

The input variables are divided into 4 categories of characteristics: TE element, TE Module, Heat exchanger and Operating conditions

The TE material has 3 properties which are a function of their mean temperature tm

Ro_Te(tm) Electrical resistivity Ω*m
S_Te (tm) Seebeck coefficient V/K
k_Te (tm) Thermal conductivity W/(m*K)
The functions used are those given in paragraph 3

5.2 TE module characteristics.

There are only two input variables that characterise he module
Nb_Te= Number of TE elements in TE module
GF = Geometric factor: (A_Te/L_Te)

5.3 Heat Exchanger Characteristics.

There are 5 variables
Rt_Ba_Co = thermal **R**esistance of **C**ooled **B**ase
Rt_Hy_Co = thermal **H**ydraulic **R**esistance of **C**ooled fluid
Rt_Ba_He = thermal **R**esistance of **He**ated **B**ase
Rt_Hy_He = thermal **H**ydraulic **R**esistance of **He**ated fluid
C_xt = External thermal Conductance

.4 Operating Conditions

There are 3 variables:

	A	Electrical current **i**
_FL_Co	°C	temperature of **C**ooled **F**luid out of which heat is pumped
_FL_He	°C	temperature of **He**ated **F**luid into which heat is dissipated

7. RESOLUTION

We have a system of 6 linear equations etween 6 unknown variables :

x1 = P_Co
x2 = P_He
x3 = t_Ce_Co
x4 = t_Ce_He
x5 = t_Ba_Co
x6 = t_Ba_He

rranging the 6 equations with respect to the 6 nknown variables we have the following system:

1 *x1 + a12 *x2 + a13 *x3 ....+a16 *x6 = b1
21 *x1 + a22 *x2 + a23 *x3 +..........+a26 *x6 = b2
31 *x1 +......................................+a36 *x6 = b3
41 *x1 +......................................+a46 *x6 = b4
51 *x1 +......................................+a56 *x6 = b5
61 *x1 +......................................+a66 *x6 = b6
ith :

a11 = a22 = a33 = a44 =a55 = a66 = 1
a13 = +S_Mo * i + C_Mo
a14 = a23 - C_Mo
a15 = a26 = C_xt
a16 = a25 = - C_xt
a24 = - S_Mo * i + C_Mo
a31 = - Rt_Ba_Co -Rt_Hy_Co
a42 = - Rt_Ba_He -Rt_Hy_He
a62 = - Rt_Hy_He

b1 = - S_Mo * i * 273 + (Re_Mo * i^2)/2
b2 = + S_Mo * i * 273 + (Re_Mo * i^2)/2
b3 = b5 = t_FL_Co
b4 = b6 = t_FL_He
all the other terms are equal to zero

There are 11 input variables implied in the matrix : C_Mo, C_xt, i, Re_Mo, Rt_Ba_Co, Rt_Ba_He, Rt_Hy_Co, Rt_Hy_He, S_Mo, t_FL_Co, t_FL_He

The resolution of the classical matrix multiplication

A * X = B gives :

P_Co = x1
P_He = x2
t_Ce_Co = x3
t_Ce_He = x4
t_Ba_Co = x5
t_BA_He = x6

8. SPREADSHEET SOLVING

We have used Excel 4.0 on the Macintosh, it is the same with Excel 4.0 on MS Windows for the PC, but one can use any spreadsheet that has matrix inversion and preferably convergence by iteration.

Advantages of using a spreadsheet: Solving with a spreadsheet is simpler, easier and quicker than writing a program in one of the languages BASIC, C, FORTRAN or Pascal etc.

There are other advantages:

- it is easy to generate data tables with one or two inputs.
- it is easier to develop graphs etc. because the data base for the files exists and it is easy to connect to other programs (import and export)
- compact and easy to read presentation compared to a program, whatever the language

We have chosen to fill in two columns and then 6 columns for the matrix
Column A is for titles. We have chosen the name of the formula in the cell (col. B) to be identical to the name in the cell on the left (col. A).

9. PRESENTATION OF THE WORKSHEET

The worksheet with the equations is given in Appendix 1.
Column A contains the titles and names of the variables, column B contains numerical values or equations.
Lines 1 to 22(col. A and B) contain the **System characteristics** and the **Operating conditions.**
Lines 5,6 and 7 are the TE element characteristics as a function of the average temperature tm of the TE element.
Lines 24 to 28 (col. A and B) are **Precalculations** (average temperature of the TE material and the characteristics of the TE module).

corresponding to the set of 6 simultaneous equations.
Lines 41 to 47 (col. A and B) are the terms of **Vector B**
Lines 49 to 54(col. A and B) is the Solution of **A.*X=B**
Line 56 (col. A and B) is the calculated average tempera-ure of the TE element tm_Te2
Lines 58 to 60 (col. A and B) gives the final output: elec-rical power, voltage across the module and COP

10. CALCULATION PROCESS

One inputs all the System characteristics col. B lines to 22

The worksheet has a circular reference formula ine 24: **tm_Te = Tm_Te2** which is defined on line 56 ut Excel has the function iteration, which solves this au-omatically by iterating. There is the choice of defining ne number of iterations, or of defining the maximum hange between iterations.

We have found that after 3 iterations **tm** only hanges by a fraction of a degree C so this gives an ac-uracy of better than 1%. If one fixes the maximum hange between iterations one can choose 0.1 °C.The orksheet accomplishes the 3 iterations within a second r two.

The equations we have used give negative cooling owers and COP. In generating the tables we have hanged their signs.

11. EXAMPLES

We have chosen the Melcor module CP5-31-06, the aterial properties are given in paragraph 3. This odule has 62 elements and the geometric factor GF is ual to 0.012 m. The size of the ceramic is 55*55 mm it this only affects the thermal resistance of the base.

e have examined two cases

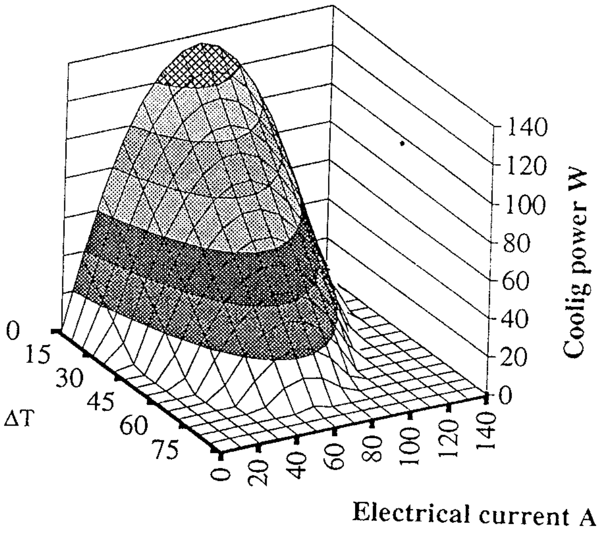
1) one of a module without heat exchangers.
2) one of a module with on the cooled side an in-face resistance corresponding to thermal grease and on heated side a heat sink.

e results are presented as 3 dimensional graphs. The is's are in the horizontal plane: the electrical current d the difference in temperature between the hot side 7 °C) and the cold side. The vertical axis is the cooling wer or the COP.

. 2 is the module alone, the vertical axis is the cooling wer. Fig. 3 gives the COP also for the module alone.

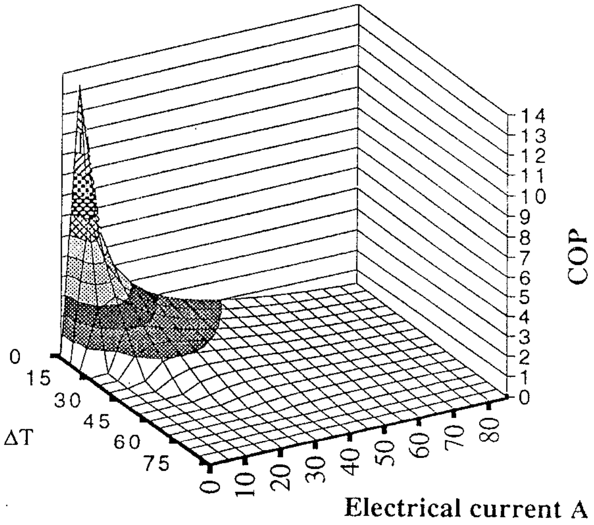
e graphs of Fig. 2 and 3 correspond to the perfor-ances given by the module manufacturer, the ΔT is the mperature difference between the ceramic on the heated de and the ceramic on the cooled side.

g. 4 is cooling power of the module with a heat sink d Fig. 5 the corresponding COP where the ΔT is tween a cooled plate (of no thermal resistance but with small thermal interface resistance between the plate and cooled ceramic) and heated gas (air) through a heat k with a thermal resistance.



Temperature of heated ceramic = 27 °C
ΔT = temp.of heated ceramic - temp. of cooled ceramic

Fig. 2 Cooling power of a Melcor CP5-31-06 module alone versus electrical current and the ΔT across the module



Temperature of heated ceramic = 27 °C
ΔT = temp. of heated ceramic - temp. of cooled ceramic

Fig. 3 COP of a Melcor CP5-31-06 module alone versus electrical current and the ΔT across the module

The spreadsheet calculates the ceramic temperature on the heated side, there is a large temperature difference between the heated side ceramic temperature and the heated fluid temperature, this difference explains the considerable drop in performance. This difference is often underestimated by users of TE modules and this leads to cooling powers well below those of a module on its own.

nd example are on the cooled side no hydraulic esistance

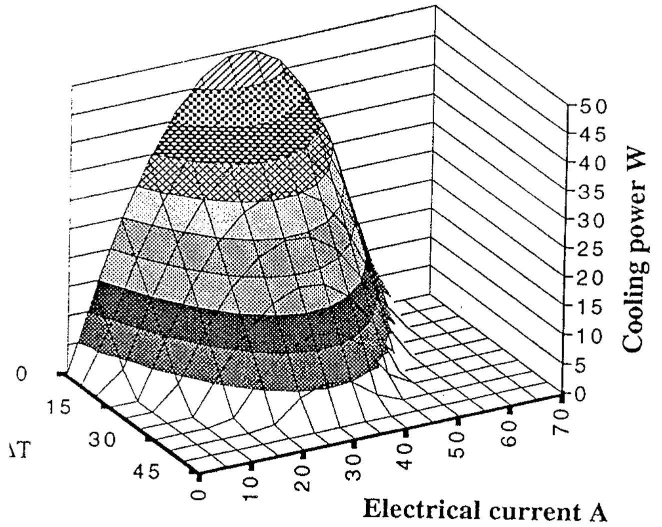
t_Hy_Co = 0

t_Ba-Co= 0.13 K/W this corresponds to a thermal rease with an interface resistivity of 0.35 K*cm^2/W and e Melcor module CP5-31-06 with ceramic size of 55*55 m. On the heated side they correspond to those of a mpact air cooled heat sink

t_Ba_He = 0.045 K/W this corresponds to a base of *90 *5 mm.

t_Hy_He = 0.25 K/W this is an average value for a com- ct air heat exchanger with fins and forced convection.

_xt = 0.01 K/W this corresponds to the heat losses rough a thermal insulation material with some losses rough screws.



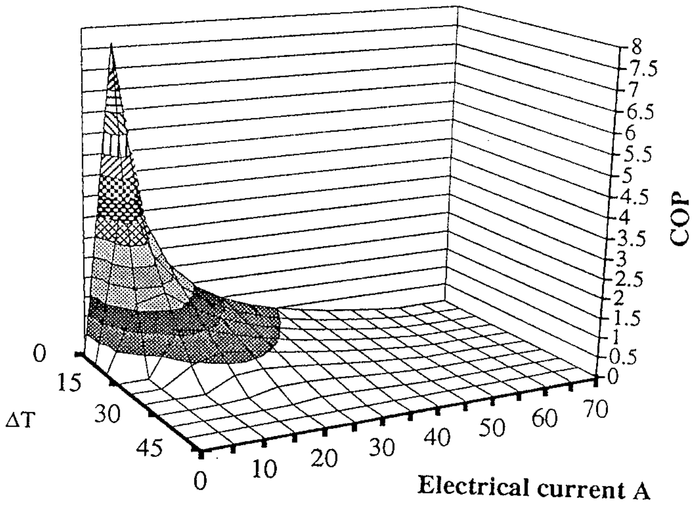
mp. of heated fluid = 27 °C

T = temp.of heated fluid - temp. of cooled plate

g. 4 Cooling power of a Melcor CP5-31-06 module th a heat sink versus electrical current and the ΔT tween the cooled plate and the heated fluid.

We note that the two graphs for the cooling have e same shape and the two sets of graphs for the COP ve the same shape but the numerical values are very fferent. What this shows is that for this Melcor module P5-31-06 alone the maximum cooling is obtained with electrical current of 70 A.

UT with a compact heat sink the maximum cooling is tained with an electrical current of 30 A. Between this odule alone and this one with a heat sink, the values of oling power and COP are decreased by about 50%. So user must be careful not to use a current that exceeds value corresponding to the maximum cooling for a ven heat exchanger. It is important to underline that en volume is available and one wants to get the ximum cooling from a module, the heat sink must be nensioned in consequence. One can dimension the heat changer so that the maximum cooling is obtained with a rrent just below 70 A.



Temp. of heated fluid = 27 °C

ΔT = temp.of heated fluid - temp. of cooled plate

Fig. 5 COP of a Melcor CP5-31-06 module with a heat sink versus electrical current and the ΔT between the cooled plate and the heated fluid.

12. CONCLUSIONS

A simple method using a worksheet to calculate the performances of a thermoelectric module has been presented. The cooling power and the COP of a module alone and those of a module with a heat sink are given as three dimensional graphs as a function of electrical current and the difference in temperature between the heated side and the cooled side. We see for a given module the tremendous influence of the heat sink on the cooling power and the COP.

References:
1. Goldsmid H.J. Electronic Refrigeration Pion Ltd. London 1986
2. Schlicklin P. M., Stockholm J. G. Thermoelectric module characterisation. 7th International Conference on Thermoelectric Energy Conversion. Arlington Texas March 1988.

Excel is a Registered trademark of Microsoft Corporation

	A	B
1	**Mathematical Model**	Thermoelectric module with heat exchangers
2	Version : 10 September 1992	Excel 4.0 worksheet
3	**SYSTEM CHARACTERISTICS**	
4	**TE element characteristics**	**Melcor SN1**
5	S_Te	=(210.9019+0.34426*(tm_Te-23)-0.0009904*(tm_Te)^2)/1000000
6	Ro_Te	=(10.8497+0.0535*(tm_Te-23)+0.0000628*(tm_Te-23)^2)/1000000
7	k_Te	=1.65901-0.00332*(tm_Te-23)+0.0000413*(tm_Te-23)^2
8	**TE module**	
9	**Characteristics**	**CP5-31-065**
10	Nb_Te	62
11	GF	0.012
12	**Heat Exchanger**	
13	**characteristics**	
14	Rt_Ba_Co	0
15	Rt_Hy_Co	0.013
16	Rt_Ba_He	0.045
17	Rt_Hy_He	0.25
18	C_xt	0.01
19	**OPERATING CONDITIONS**	
20	i	40
21	t_FL_Co	-3
22	t_FL_He	27
23	**PRECALCULATIONS**	
24	tm_Te	=tm_Te2
25		
26	S_Mo	=Nb_Te*S_Te
27	Re_Mo	=Nb_Te*Ro_Te/GF
28	C_Mo	=Nb_Te*k_Te*GF

29	A	B	C	D	E	F	G
30	**Calculations**						
31	**Matrix A.**						
32		**P_Co**	**P_He**	**t_Ce_Co**	**t_Ce_He**	**t-Ba_Co**	**t_Ba_He**
33	**aij**	**ai1**	**ai2**	**ai3**	**ai4**	**ai5**	**ai6**
34	a1j	1	0	=S_Mo*i+C_Mo	=-C_Mo	=C_xt	=-C_xt
35	a2j	0	1	=-C_Mo	=-S_Mo*i+C_Mo	=-C_xt	=C_xt
36	a3j	=-Rt_Ba_Co-Rt_Hy_Co	0	1	0	0	0
37	a4j	0	=-Rt_Ba_He-Rt_Hy_He	0	1	0	0
38	a5j	=-Rt_Hy_Co	0	0	0	1	0
39	a6j	0	=-Rt_Hy_He	0	0	0	1

40		
41	**Vector B.**	
42	b1	=-S_Mo*i*273+(Re_Mo*i^2)/2
43	b2	=+S_Mo*i*273+(Re_Mo*i^2)/2
44	b3	=t_FL_Co
45	b4	=t_FL_He
46	b5	=t_FL_Co
47	b6	=t_FL_He
48	**Solution of A.*X=B.**	
49	P_Co	=MMULT(MINVERSE(A.),B.)
50	P_He	=MMULT(MINVERSE(A.),B.)
51	t_Ce_Co	=MMULT(MINVERSE(A.),B.)
52	t_Ce_He	=MMULT(MINVERSE(A.),B.)
53	t_Ba_Co	=MMULT(MINVERSE(A.),B.)
54	t_Ba_He	=MMULT(MINVERSE(A.),B.)
55		
56	tm_Te2	=(t_Ce_Co+t_Ce_He)/2
57	**Final output**	
58	P_eL	=P_He-P_Co
59	U	=P_eL/i
60	COP	=P_Co/P_eL