1 - Large scale thermoelectric cooling. J. G. Stockholm, J. P. Despres. 2 nd International Conference on Thermoelectric Energy Conversion. Arlington Texas , 22-24 March 1978. The University of Texas at Arlington, Arlington Texas 76019 , Dept of Electrical Engineering Editor K.R. Rao P.O. Box 19016) pages 54-59 IEEE Catalog number 78CH1313-6 REG.5

LARGE SCALE THERMOELECTRIC COOLING

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Summary :
A mathematical model is presented for gas to gas cooling. A brief comparison of counter flow and cross flow systems leads to the choice of the latter. The performances along the gas flows trajectory are given per surface area of thermoelectric material. The influence of various parameters such as the coefficient of merit of the T.E. material are examined and two cooling units are presented with their performances.

Thermoelectric cooling units which are heat pumps are analogous to heat exchangers where an active material (the thermoelectric material abreviated T.E.) creates heat and cooling between the two sides of the heat exchanger. It is therefore possible to classify T.E. cooling like heat exchangers by the relative movement of the two fluid flows.

- parallel flow (both flows in the same direction).
- parallel counter flow.
- cross flow.

The first of these three is excluded as theoretically always less efficient than counter flow, but the latter and cross flow each have their respective advantages.

Two mathematical models were developped but only the cross flow will be presented here, the other has the same equations, only the numerical calculating method differs. After the presentation of the model a comparison with counter flow will be given.

1. Mathematical model

In large thermoelectric cooling systems, the many thermoelectric junctions operate under different conditions. It is necessary to have a mathematical model that can calculate, with given inlet conditions (temperatures and flow rates), the operating conditions at each junction or set of junctions.

The model here described is for cross flow as shown in Figure 1.

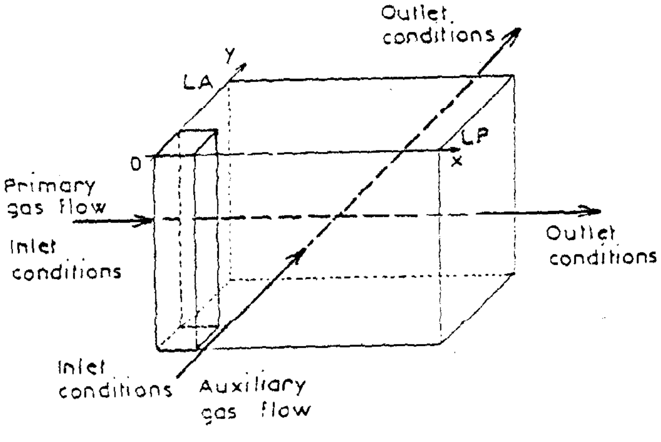

Figure 1 - Schematic of cross flow exchanger.

The step by step calculation is done by starting at x = o and increasing y up to the value LA (length of auxiliary circuit). Then the calculation starts at e.g. x = 1 and does y = o to y = LA and so on, till x = LP and y = LA is reached.

The model must go from gas conditions on primary circuit to gas conditions on secondary circuit. The basic thermoelectric equations are very simple and well known.

$$\left.\begin{aligned} P_c &= aIT_c + \frac{1}{2} RI^2 + C\,(T_h - T_c) + C_S\,(T_{BH} - T_{BC}) \\ P_H &= aIT_H + \frac{1}{2} RI^2 + C\,(T_H - T_c) + C_S\,(T_{BH} - T_{BC}) \end{aligned}\right\} \quad (1)$$

$$P_E = aI\,(T_H - T_C) + RI^2$$

Equations (1) are the energy equations across the thermoelectric material, from (1), equations must be established that give the gas temperatures. It is necessary to introduce the following parameters :

- thermal resistance of heat exchangers on either side of the thermoelectric material.
- thermal resistance between bases excluding the thermal resistance of the thermoelectric material C_s^{-1}).
- electrical conductivity of all circuits between each thermoelectric junction.

Each of these parameters require calculations and experimental data to guarantee their validity.

The curves that are given in the following paragraphs were obtained with this model, a print out is given in Figure 2. (overall and partial).

```
RESULTATS TOTAUX
----------------
          PUISSANCE FRIGORIFIQUE THERMOELEMENT       -6103.00 W
          PUISSANCE FRIGORIFIQUE TOTALE              -5083.00 W  KW
          PUISSANCE CALORIFIQUE THERMOELEMENT        10563.77 W
          PUISSANCE CALORIFIQUE TOTALE                9645.84 W
          PUISSANCE DE L ISOLANT                       966.54 W
          PUISSANCE ELECTRIQUE (PC-PF)                4562.84 W
          PUISSANCE ELECTRIQUE (U*I)                  4574.67 W
          TENSION ELECTR. D ALIMENTATION(PC-PF)/I      30.419 V
          TENSION ELECTR. D ALIM.(R*I+ALFA*DT)         30.498 V
          FORCE CONTREELECTROM. PELTIER(ALFA*DT)        6.725 V
          RENDEMENT FRIGORIFIQUE                 1.114
            DT  MOYEN   ECHANGEUR COTE CHAUD            4.04 DEG.C
            DT  MOYEN   ECHANGEUR COTE FROID            1.68 DEG.C
            DT  MOYEN ENTRE FACES THERMOELEMENT        22.73 DEG.C
          TEMP. MOYENNE FACE CHAUDE THERMOELEMENT       39.21 DEG.C
          RESISTANCE ELECTRIQUE DES CONNEXIONS     0.3839994E-03 OHM
          RESISTANCE ELECTRIQUE DES COLONNES       0.1584836E 00 OHM
          RESISTANCE ELECTRIQUE DE LA BATTERIE     0.1588675E 00 OHM
          PUISSANCE ELECTRIQUE TOTALE             4571.48 W
          TEMP. MOYENNE SORTIE AIR FROID              12.32     DEG.C.
          HUMIDITE                                   100.05
          TEMP. MOY. SORTIE AIR CHAUD            37.01    DEG.C.
          HUMIDITE                                    29.81
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Figure 2 - Computer output
Overall results

```
RANG NO   32
NAPPE             AIR A REFROIDIR                                      AIR A RECHAUFFER            DT ENTRE          T E M P E R A T U R E
 NO   TEMPERATURE  HUMIDITE  PUISSANCE  TEMPERATURE  HUMIDITE  PUISSANCE     FACES     FACE CHAUDE  FACE FROIDE  SURFACE AIL.
        DEG.C.      P.CENT       W         DEG.C.     P.CENT       W         DEG.C.       DEG.C.      DEG.C.        DEG.C.
  1     11.83      100.02      -2.67       32.00      40.00      5.66       25.19        36.12       10.93         11.15
  2     12.11      100.09      -2.58       32.94      37.84      5.60       25.79        37.01       11.23         11.49
  3     12.39      100.07      -2.51       33.87      35.82      5.54       26.33        37.90       11.56         11.79
  4     12.67       99.89      -2.42       34.79      33.93      5.48       26.91        38.77       11.86         12.08
  5     12.93       99.95      -2.34       35.70      32.17      5.42       27.48        39.64       12.15         12.37
  6     13.18       99.98      -2.26       36.60      30.53      5.36       28.05        40.49       12.45         12.66

SORTIE
 BLOC                                                                                   37.50         28.99

           CONDENSATION ENTRE LES NAPPES NO   1 ET   6
            PUISSANCE FRIGORIFIQUE THERMOELEMENT         -155.27 W
            PUISSANCE FRIGORIFIQUE TOTALE                -118.21 W
            PUISSANCE CALORIFIQUE THERMOELEMENT           298.34 W
            PUISSANCE CALORIFIQUE TOTALE                  264.48 W
            PUISSANCE DE L ISOLANT                         35.39 W
            PUISSANCE ELECTRIQUE                          146.26 W
            TENSION ELECTRIQUE D ALIMENTATION              0.975 V
            RENDEMENT FRIGORIFIQUE                  0.808
              DT  MOYEN   ECHANGEUR COTE CHAUD            3.55 DEG.C
              DT  MOYEN   ECHANGEUR COTE FROID            0.72 DEG.C
              DT  MOYEN ENTRE FACES THERMOELEMENT        26.63 DEG.C
            TEMP. MOYENNE FACE CHAUDE THERMOELEMENT       38.32 DEG.C
```

Figure 2 - Computer output. Conditions at exit bloc x = LP. Nappe N° is y direction.

A completely dimensionless presentation is ideal for comparisons between various results. The authors objective here is to enable those interested in the possibilities of thermoelectric cooling to be able to estimate a system hence the following units have been used :

- Cooling power is systematically given per unit surface of thermoelectric material employed per meter length of system.

- Primary mass flow rate per surface of thermoelectric material per meter length of exchanger =

$\mathrm{kg.s^{-1}}$ ($\mathrm{cm^{-2}}$ of $\mathrm{TE.m^{-1}}$ length)

- Auxilary mass flow.

In the case of counter flow it can be given as the following ratio :

$$\bar{Q} = \frac{\text{auxilary mass flow}}{\text{primary mass flow}}$$

In the case of cross flow it is logical to give it as :

$$\bar{Q} = \frac{\text{auxilary mass flow per meter length of primary circuit}}{\text{primary mass flow}}$$

1.1. Comparison between counter flow and cross flow

A comparison between counter flow and cross flow is given in Fig. 3. The units are those indicated above.

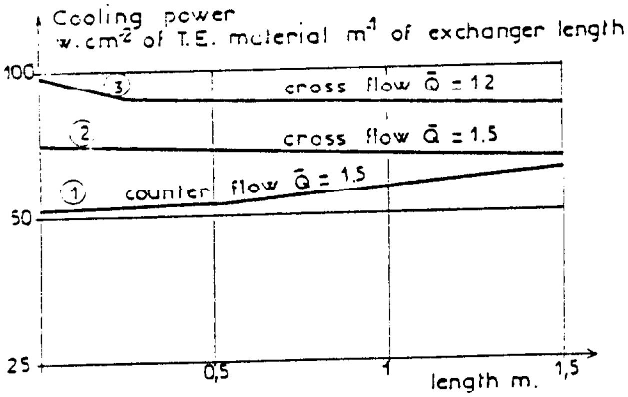

Fig. 3 - Cooling power along T.E. heat exchanger.

Primary flow = $5.2\,10^{-4}$ $\mathrm{kg.s^{-1}}$ ($\mathrm{cm^{-2}}$ TE $\mathrm{m^{-1}}$)

$$\bar{Q} = \frac{\text{auxilary flow}}{\text{primary flow}}$$

T.E. material $Z = 2.34.10^{-3}\ \mathrm{K^{-1}}$

Current density $J = 133\ \mathrm{A\ cm^{-2}}$

Thermal resistance (primary + auxilary) = $0.88\ \mathrm{K.W^{-1}}$

Curve 1 is counter flow.

Curve 2 is cross flow.

All the physical characteristics of the exchangers and the operating conditions are the same.

- Inlet temperatures
- Primary flow
- Ratio auxilary flow to primary flow
- Current density
- Heat transfer coefficients
- Thermal resistances.

Curve 3 is cross flow where the gas velocity of the auxilary flow is the same as for the counter flow curve 1.

Cross flow and counter flow exchangers can be directly juxtaposed on the primary circuit.

In the case of counter flow both circuits will be lengthened hence reducing the cooling poser. It is possible at the junction to exit the auxilary gas from the first unit and introduce new gas for the second unit, but this is complicated technology wise.

In the case of cross flow the juxtaposition only lengthens the primary circuit and the auxilary circuit creates no technology problems.

Cross flow exchangers are now examined in detail.

1.2. Cross flow exchangers

The major parameter in the design of cross flow exchangers is the ratio :

$$\frac{\text{LP (length of primary gas flow x direction}}{\text{LA (length of auxilary gas flow y direction}}$$

A comparison has been done and given in Figure 4 below.

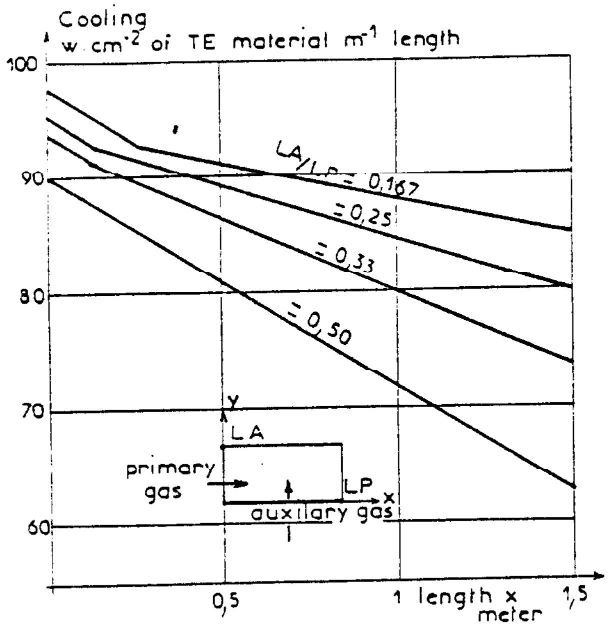

Figure 4 - Cross flow thermoelectric heat exchanger

Cooling power along heat exchanger length.

Parameters :

primary gas flow $5.2.10^{-4}$ kg.s^{-1} (cm^{-2} TE.m^{-1})

$\frac{\text{auxilary flow per meter}}{\text{primary flow}} = 8$

T.E. material $Z = 2.34.10^{-3}$ K^{-1}

J. current density 133.A.cm^{-2}

The following parameters remained unchanged :

- Thermoelectric material
- Current density
- Inlet temperatures
- Primary mass flow
- Ratio auxilary mass flow to primary mass flow.

The only parameter that is changed is LA length of primary gas circuit. For presentation reasons the curves are parametered with the ratio $\frac{LA}{LP}$. It must be noted that since :

- LA is varied and
  primary mass flow is constant

the gas velocity in the primary circuit varies.

It is also interesting to know how the cooling power varies as a function of y (LA direction) for different values of x (LP direction). The cooling power as a function of LA is given in Figure 5, where the curves are parametered for 4 values of x (primary air flow direction). The cooling powers decrease as x and y increase, which means that the smaller the system the more power one can obtain for a given surface of thermoelectric material.

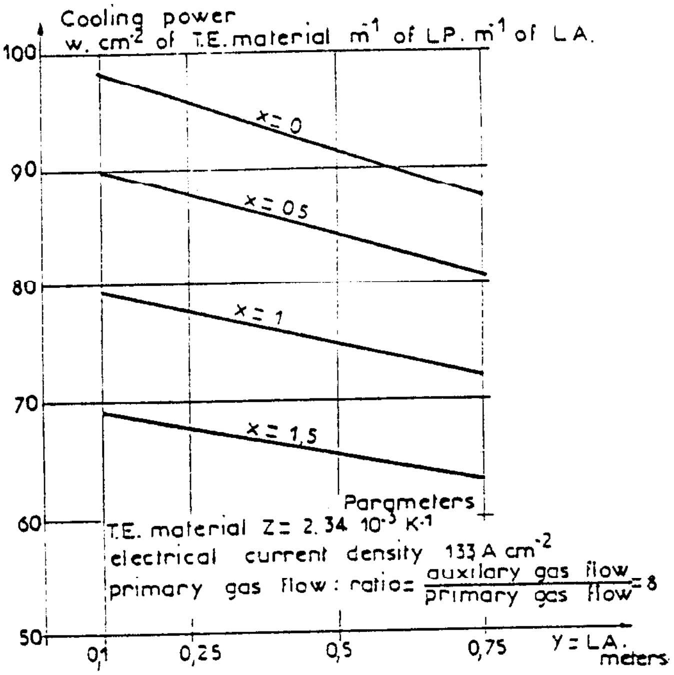

Figure 5 - Cooling power along auxilary axis.

2. Influence of various parameters

2.1. Material characteristics

The parameters that intervene are a : Seebeck coefficient, ρ electrical resistivity, k thermal conductivity. The coefficient of merit $Z = a^2 \rho^{-1} k^{-1}$ which is used to compare thermoelectric materials can be introduced and Fig. 6 gives the cooling power and coefficient of performance C.O.P. as a function of Z.

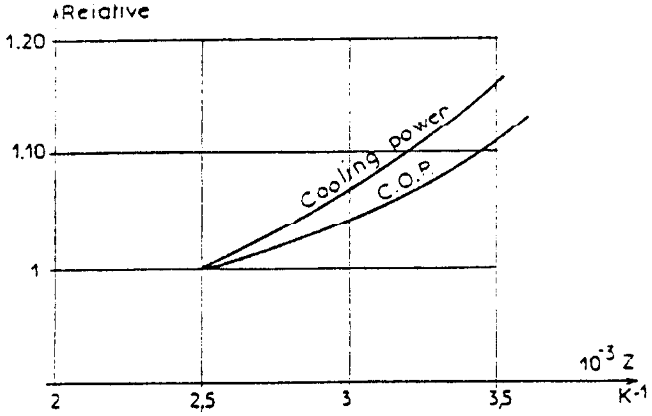

Figure 6 - Influence of Z on relative cooling power and C.O.P.

These curves depend on the values of a, ρ, k, J (electrical current density), ($T_h - T_c$). Nevertheless for large systems when Z goes from $2.5.10^{-3}$ K^{-1} to $3.5.10^{-3}$ K^{-1} the increase in cooling power is of the order of 15 % which is under half of the relative increase in Z.

The thickness of the thermoelectric material influences the cooling power. Figure 7 indicates the relative cooling power with the thickness varying from 0.7 to 2.5 mm.

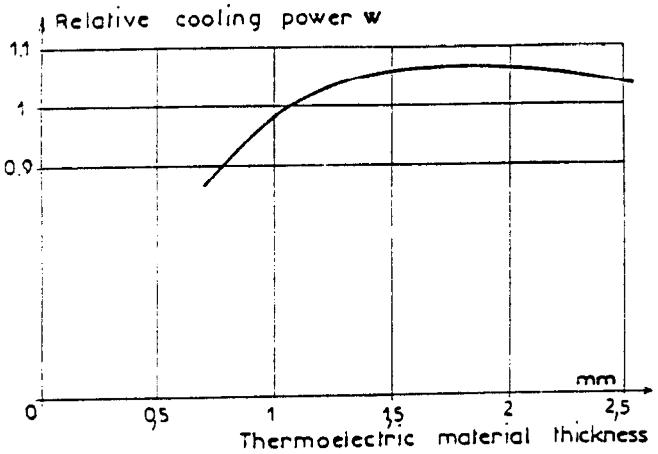

Figure 7 - Influence of thermoelectric material thickness on cooling power.

2.2. Thermal resistances

The overall performances of a thermoelectric system depend tremendously on the thermal resistances.

The thermal resistance between gas and thermoelectric surface can be written for either side of the T.E. material :

$$(2) \quad \frac{I}{C} = \frac{I}{C_B} + \frac{I}{H \measuredangle} \qquad \text{(see notations)}$$

where the thermal resistance is divided into parts one which goes from T.E. surface to base of fins which is essentially a thermal heat conduction problem, C_B^{-1} varies generally between 0.2 WK^{-1} and 0.5 WK^{-1} the other goes from the base of the fins to the gas

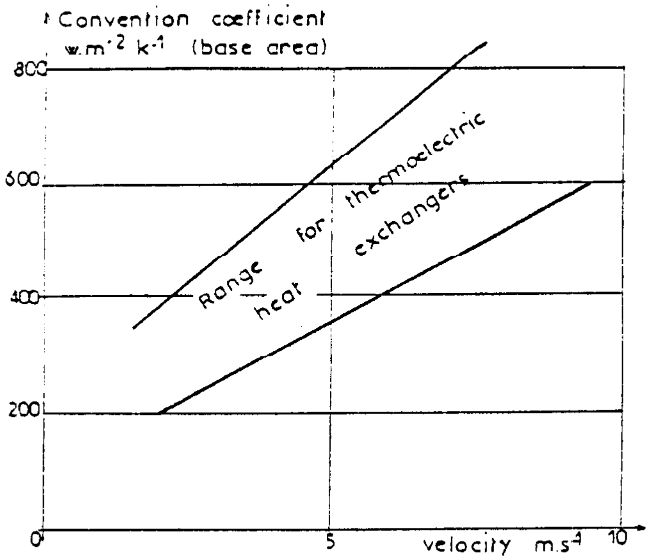

Figure 8 - Range of convection coefficient per base area as a function of gas velocity.

The above figure gives the range of the convection coefficient H per area of base for thermoelectric heat exchanger as a function of gas velocity.

Another thermal resistance which must not be neglected is the one between cold side exchanger and hot side exchanger $(C_{S_1} \cdot \measuredangle)^{-1}$, C_S is usually between 5 and 20 $W.m^{-2}\ K^{-1}$

3. System design

A thermoelectric cooling system can be designed in two stages which are relatively independent.

The first stage is to know approximatively how the performances vary inside the system along both gas flows. Fig. 4 gives an approach to the problem. It is then necessary to evaluate the average heat exchanger in the system, or consider several "average" heat exchangers if the operating conditions are very different from one end to the other of the system.

3.1. Dimensioning of average heat exchanger

Having the required characteristics of an exchanger it is possible to study it in detail using equation (1) and introducing equation (2) the latter enables the introduction of experimental data concerning the base thermal conductivity C_B and H the convection coefficient per exchanger base area.

The coefficient of performance must include the power used by the two gas flows.

$$Pa = \frac{q_c \Delta p_c}{\eta_c} + \frac{q_H \Delta p_c}{\eta_H}$$

The ranges of cooling power and C.O.P. (including fan power) are given for an industrial gas to gas thermoelectric heat exchanger in Fig. 9.

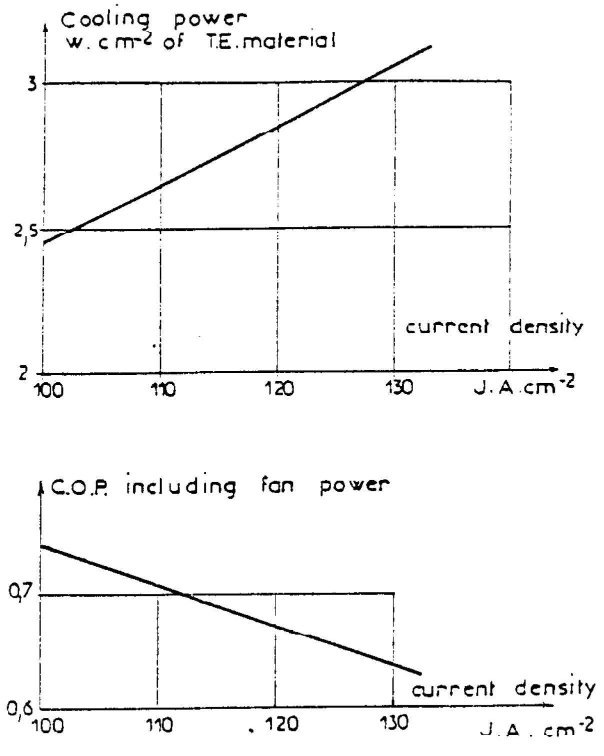

Figure 9 - Cooling power and C.O.P. including fan power, for average heat exchanger.

3.2. Design of a modular system

The average heat exchangers having been optimized it is possible to calculate the whole system.

The most convenient parameter in the thermoelectric energy equation is the intensity because it is an independent parameter. Certain measured values are entered into the mathematical model.

- Properties of the thermoelectric material, they are checked in the laboratory.
- Total electrical resistance of the system is measured on the unit.
- The thermal properties of all materials used, where there is a temperature difference, are measured experimentally.

The prototype units are laboratory tested with an on line small computer to obtain during the actual test the overall heat balances. The experimental results are compared with the model, the accuracy of the model is ± 3 %. Figure 10 gives the cooling power of a small unit as a function of current density.

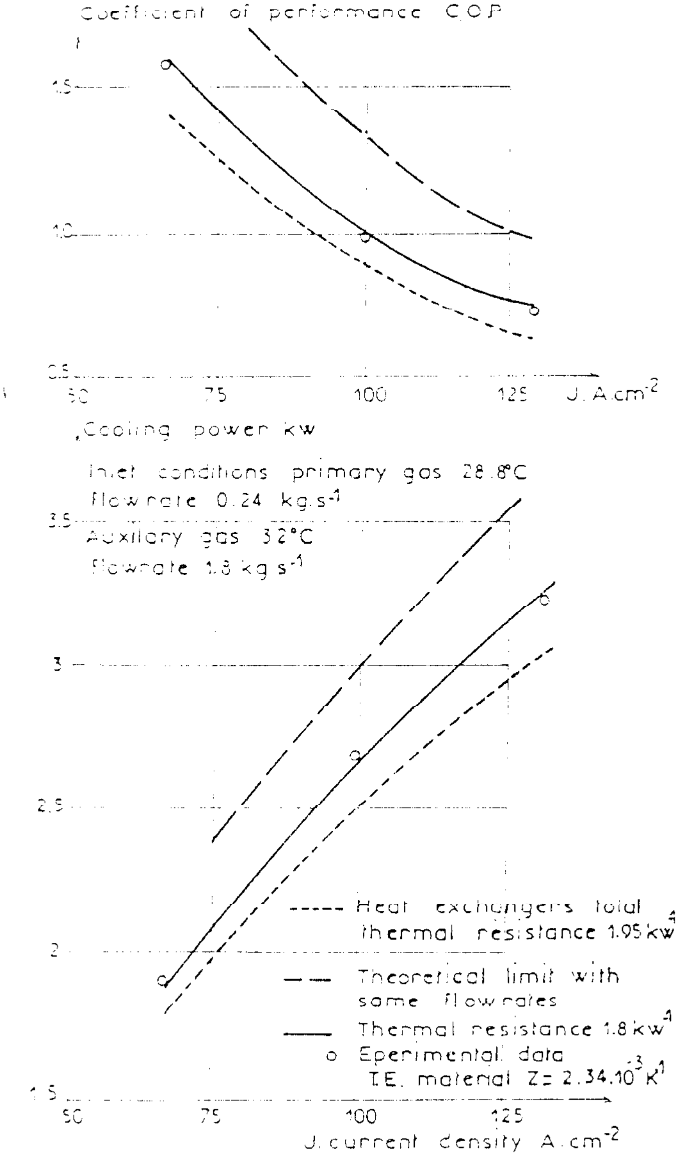

Fig. 10 - Performances of industrial unit.

This unit would normally be operated at a current density between 100 and 125 $A.cm^{-2}$, it cools a gas flow of 0.24 $kg.s^{-1}$ with an inlet temperature of 28.8° C. The graph has three curves. The two lower ones (continuous line and short dash) correspond to two different values of the heat exchangers thermal resistance (1.8 $K.W^{-1}$ and 1.95 $K.W^{-1}$). The long dashed curve corresponds to an exchanger where the only thermal resistance would be from the convection coefficient of the fins.

A photograph of the unit tested in the laboratory is given in Figure 11.

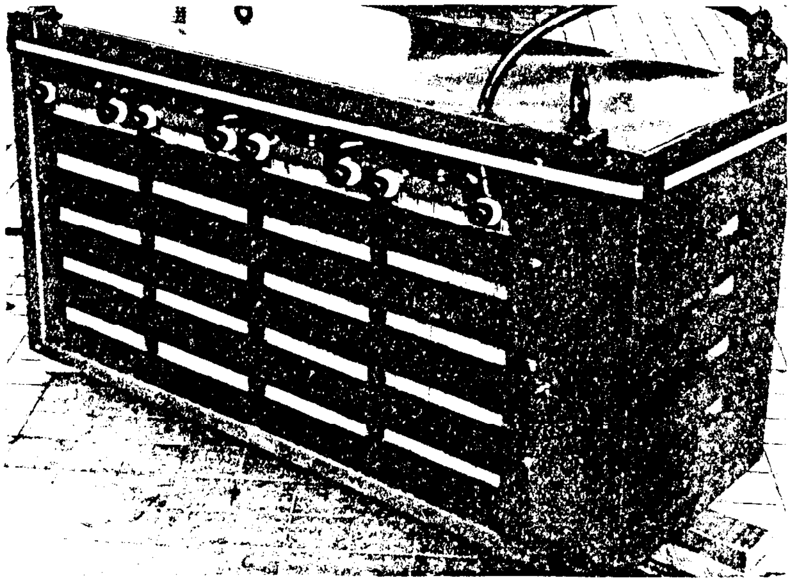

Figure 11 - Photograph of a laboratory prototype of 3 kW of cooling.

Such a unit is an assembly of 4 subunits like the one shown below.

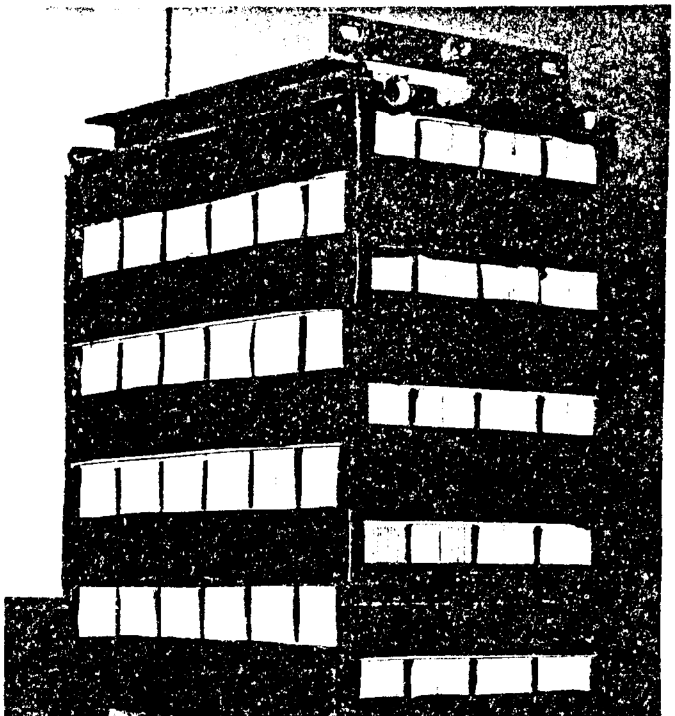

Fig. 12 - Industrial subunit.

The performances of a large size unit are given below.

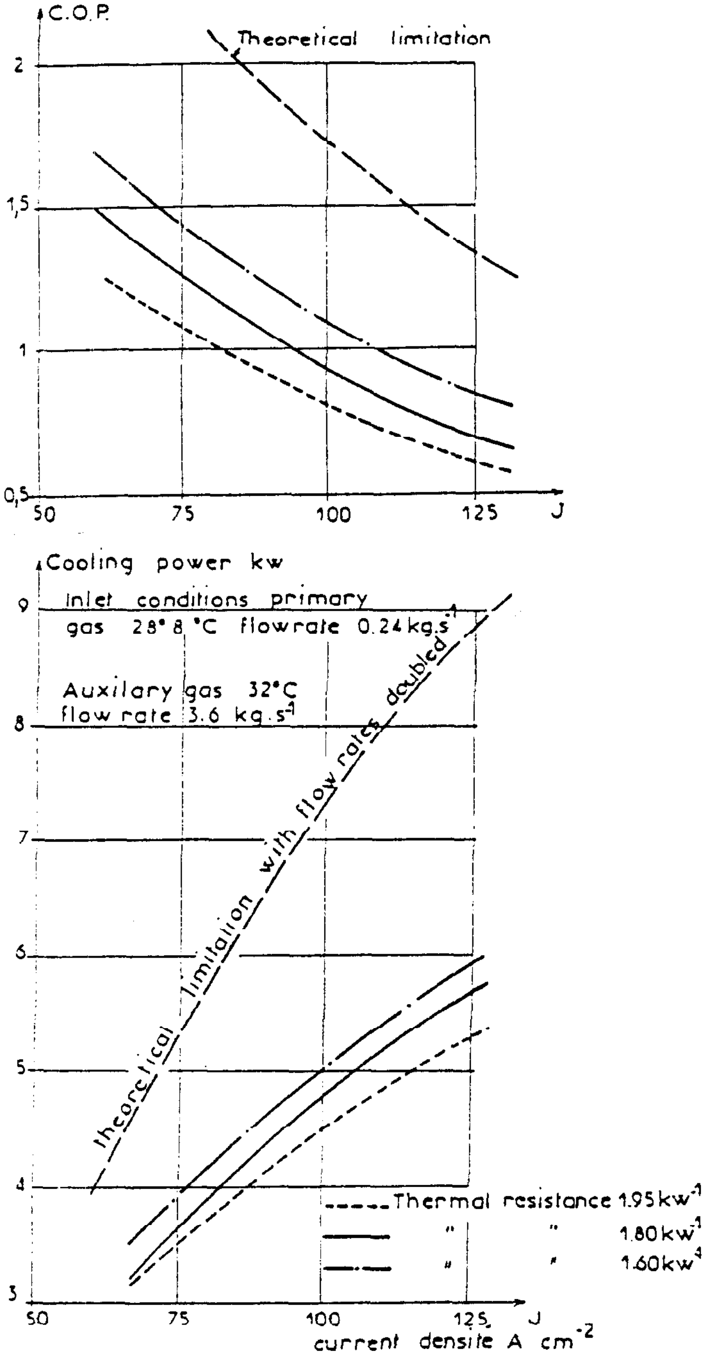

Fig. 13 - Cooling power and C.O.P. of large industrial unit.

It consists of two units of Figure 10 juxtaposed on the primary gas circuit. The only operating parameter that changes is that the auxiliary gas flow is doubled. For a current density of 100 $A.cm^{-2}$, the cooling power increases by only 76 % and the C.O.P. drops from 1.0 to 0.95.

4. Conclusions

The design of large scale thermoelectric cooling units requires the use of a mathematical model and a great deal of experimental data.

The quantity of thermoelectric material employed is a predominant factor ; when increased with the fin area the performances will increase but also the size and the cost of the installation. The characteristics of two industrial units are given with their theoretical limitations with thermoelectric materials currently manufactured.

With better thermoelectric materials the cooling power improves by less than 50 % of the relative increase in the coefficient of merit Z.

Notations

Symbol	Units	Designation
a	$V.K^{-1}$	Seebeck coefficient
C	$W.K^{-1}$	Thermal conductance of T.E. material
C_B	$W.K^{-1}$	Thermal conductance inside base
C_S	$W.K^{-1}$	Thermal conductance between hot and cold bases (excluding T.E. material)
H	$W.m^{-2}\ K^{-1}$	Convection coefficient between gas and base of exchanger
I	A	Electrical current
J	$A.cm^{-2}$	Electrical current density
K	$W.m^{-1}\ K^{-1}$	Thermal conductivity of T.E. mat.
K	K	Degrees Kelvin
LA	m	Total length of exchanger aux. circ.
LP	m	Total length of exchanger prim.circ.
Pa	W	Fan power to drive gas
P	W	Power (cooling or heating)
q	$m^3.s^{-1}$	*Gas volume flow rate*
$\bar{Q}$	non dim.	Ratio auxiliary mass flow rate/ primary mass flow rate
R	Ω	Electrical resistance of T.E. mat.
R'	$W.K^{-1}$	Overall thermal resistance between T.E. surface and gas
X	m	Axis of primary gas flow (to be cooled)
Y	m	Axis of auxiliary gas flow
Z	K^{-1}	Coefficient of merit of T.E. mat.
Σ	m2	Base area of heat exchanger
η	non dim.	Fan efficiency
ρ	Ω.m	Electrical resistivity of T.E. mat.
Δ p	Pascals	Pressure drop across head exchanger
C.O.P.	K^{-1}	Coefficient of performance
T		Temperature of T.E. surface
Indices		
C		Cold side
H		Hot side
BH		Base hot side
BC		Base cold side

Bibliography

Ioffe, Semiconductor thermoelements and thermoelectric cooling, Infosearch, London 1957
Goldsmid, Thermoelectric Refrigeration
Heikes R. and Ure R., Thermoelectricity Science and Engineering, Interscience, New York 1961