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Modeling of Thermoelectric Cooling Systems

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55.1 Introduction

Thermoelectric cooling systems transfer thermal energy from a fluid at one temperature to thermal energy in a fluid at another temperature by using thermoelectric material and electrical power. A thermal thermoelectric model is presented of a system that consists of a number of cells referred to as thermoelectric building blocks. Each thermoelectric building block has three parts:

A quantity of thermoelectric material (thermoelectric component) through which an electrical current flows

A heat exchanger to cool a fluid (absorb heat from the fluid)

A heat exchanger to heat a fluid (exhaust heat)

55.2 Description of the Mathematical Thermal Thermoelectric Model

A system consists of:

A unit which is divided into a number of identical thermoelectric building blocks (Figure 1a)
Thermoelectric building blocks associated with a thermoelectric component and two heat exchangers (Figure 1b)

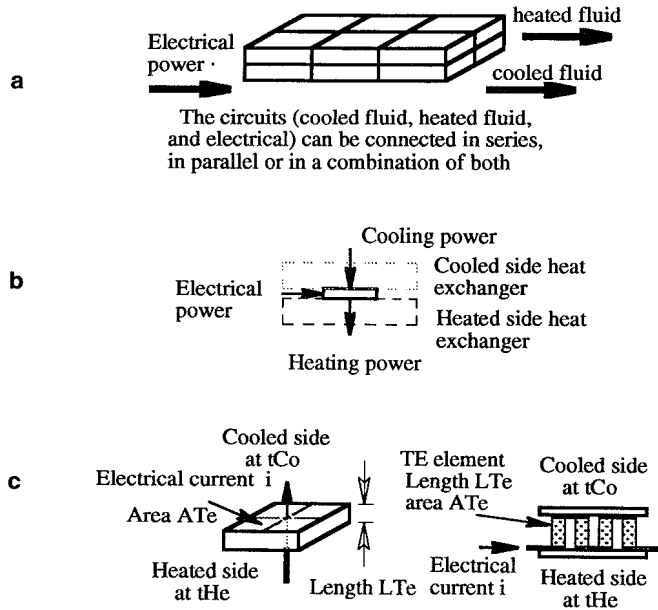


FIGURE 1 (a) Thermoelectric system with six building blocks; (b) building block power model; (c) thermoelectric element: thermoelectric module two-dimensional representation with four thermoelect elements.

Heat exchangers for gases, which can contain moisture, and for liquids

A thermoelectric component, which consists of one thermoelectric element or a thermoelect module (Figure 1c)

Thermoelectric material characteristics

55.3 Thermoelectric Material and Modules

Thermoelectric Material

Bismuth telluride is the material used for cooling and is characterized by three parameters (expressed in SI units):

- ρ_{Te} Electrical resistivity $\Omega \cdot m$
- λ_{Te} Thermal conductivity $W/(m \cdot K)$
- α_{Te} Seebeck coefficient V/K

These parameters vary with the average temperature, t_{av} , of the thermoelectric material; generally a polynomial correlation is used with second-order temperature terms. The thermoelectric material is of n- and p-type, generally the average values are used (value of n + value of p)/2.

The values depend on the manufacturer, and those used here are provided by Melcor Inc., the major world supplier of thermoelectric material and modules.

$$\rho_{Te}(t_{av}) = (10.8497 + 0.0535(t_{av} - 23) + 62.8 \cdot 10^{-6} \cdot (t_{av} - 23)^2) / 10^6$$

$$\alpha_{Te}(t_{av}) = (210.9019 + 0.34426(t_{av} - 23) - 0.9904 \cdot 10^{-3} \cdot (t_{av} - 23)^2) / 10^6$$

$$\lambda_{Te}(t_{av}) = 1.65901 - 3.32 \cdot 10^{-3}(t_{av} - 23) + 41.3 \cdot 10^{-6}(t_{av} - 23)^2$$

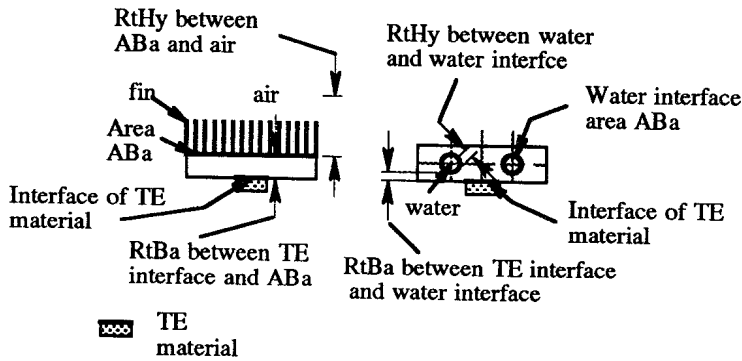


FIGURE 2 Schematic of an air heat exchanger and a water heat exchanger.

material. In addition to the material properties, the module is characterized by two other parameters: GF, the geometric factor of the thermoelectric element = A_{TE}/L_{TE} and Nb_{Te} = number of n-type elements + number of p-type elements in the module (sometimes the couple terminology is used: number of couples = $Nb_{Te}/2$).

A thermoelectric module and a single thermoelectric element can be characterized by:

$$\begin{aligned}
 Re_{Te} &= Nb_{Te} \cdot \rho_{Te}/GF = \text{total resistance } \Omega \\
 S_{Te} &= Nb_{Te} \cdot \alpha_{Te} = \text{total Seebeck } V/K \\
 C_{Te} &= Nb_{Te} \cdot GF \cdot \lambda_{Te} = \text{thermal conductance } W/K
 \end{aligned}$$

When dealing with one thermoelectric element $Nb_{Te} = 1$.

In this model the thermoelectric material characteristics are valid for a single thermoelectric element and for thermoelectric modules that contain a number of elements of n- and p-type material connected electrically in series. In the case of a thermoelectric module these characteristics include the thermal properties of the ceramic plates and of the electrical connectors, etc. This assumption is equivalent to saying that the temperature of the ceramic plate is the same as the temperature of the end of the element.

The following notations correspond either to a single thermoelectric element or to a thermoelectric module: α , the Seebeck coefficient, and C, the thermal conductance. A distinction is drawn between the electrical resistance of the cooled side and that of the heated side because when using a single thermoelectric element, the heat exchangers, which conduct the electricity between the pieces of thermoelectric material, have a non-negligible electrical resistance. This is defined as R_{Co} for the cooled side and R_{He} for the heated side.

The relevant equations are presented in Section 55.5. The terms representing Joule heating include for each side, half of the electrical resistance of the thermoelectric material plus the electrical resistance of the corresponding heat exchanger and are given by:

$$Re_{Co} = Re_{Te}/2 + R_{Co} \text{ and } Re_{He} = Re_{Te}/2 + R_{He}$$

In the case of modules, electricity is conducted from one module to the next by wires which are "dimensioned" so as to have a negligible electrical resistance, in which case $Re_{Co} = Re_{He} = Re_{Mod}/2$.

55.4 Heat Exchanger Characterization

An air heat exchanger and a water heat exchanger are shown schematically in Figure 2. When discussing the cooled side a subscript "Co" is added and when dealing with the heated side, a subscript "He." The model includes the thermal resistance of both heat exchangers and the thermal conductance (C_{s1}) of the seal and the air gap between the two heat exchangers. For simplicity,

Thermal Resistance Through a Solid

The thermal resistance through the solid, R_{tBa} (thermal base resistance, K/W):

- Water heat exchanger, it is between the interface of the thermoelectric material and the ABa in contact with the water
- Air heat exchanger, it is the thermal resistance between the interface of the thermoelectric material and the area ABa at the base on which the fins are located; it is found more convenient to use this area rather than the area of the fins because it simplifies the calculation of R_{tBa}

Contribution Due to Convection

The contribution due to convection R_{tHy} (thermal hydraulic resistance, K/W) can be expressed as $R_{tHy} = 1/(h_{Ba} \cdot A_{Ba})$ in K/W where A_{Ba} is the area of the base of the heat exchanger on the fluid side (m^2) and h_{Ba} is the convection coefficient as seen by the surface of the heat exchanger ($W/(m^2 \cdot K)$).

In the case of liquid heat exchangers the area of the liquid in contact with the base is A_{Ba} and the convection coefficient of the fluid at the interface between the fluid and the walls of the heat exchanger is h_{Ba} . An example with water is given in the Appendix.

In the case of air heat exchangers with fins R_{tHy} is calculated in the following way: the base area is A_{Ba} , the fins on the base have an area of A_{fin} , a fin efficiency of eff_{fin} and the convection coefficient of the fins is h_{fin} . Consequently $h_{Ba} = h_{fin} \cdot A_{fin} \cdot eff_{fin}/A_{Ba}$ with $eff_{fin} = (t_{fin} - t_{air})/(t_{base} - t_{air})$. An example for air is also given in the Appendix.

Thermal Conductance of a Seal

The term C_{xt} represents the thermal conductance exterior (xt) to the thermoelectric material. It includes heat conduction through the air gap between the two heat exchangers, through the heat exchanger walls and through the tightening mechanism. Experience has led the author to express this heat conductance as a function of the temperature of the bases instead of between the interface temperatures of the thermoelectric material because the average temperature of the side of the base is much closer to the base temperature than to the temperature (t_{TE}) at the ends of the thermoelectric material.

55.5 Equations for the Building Block

Equations

A set of equations for noncondensing air and for water, which correspond to the following parameters, can be written:¹

Thermal power pumped out of cooled fluid

$$P_{Co} = -S_{te} \cdot i^* (t_{TE_{Co}} + 273) + Re_{Co} \cdot \hat{i}^2 + C^* (t_{TE_{He}} - t_{TE_{Co}}) + C_{xt} \cdot (t_{Ba_{He}} - t_{Ba_{Co}})$$

Thermal power exiting the module which is heating the fluid

$$P_{He} = S_{te} \cdot i^* (t_{TE_{He}} + 273) + Re_{He} \cdot \hat{i}^2 - C^* (t_{TE_{He}} - t_{TE_{Co}}) + C_{xt} \cdot (t_{Ba_{He}} - t_{Ba_{Co}})$$

Thermoelectric material temperature in contact with cooled base

$$t_{TE_{Co}} = t_{FL_{Co}} + P_{Co} \cdot (R_{tBa_{Co}} + R_{tHy_{Co}})$$

Base temperature at interface with cooled fluid

$$tBa_{Co} = tFL_{Co} + P_{Co} \cdot RtHy_{Co} \quad (5)$$

Base temperature at interface with heated fluid

$$tBa_{He} = tFL_{He} + P_{He} \cdot RtHy_{He} \quad (6)$$

Solution

The six equations have six unknowns: tTe_{Co} , tTe_{He} , tBa_{Co} , tBa_{He} , P_{Co} , and P_{He} . The inputs consist of the operating conditions: i , tFL_{Co} , tFL_{He} ; the characteristics of the thermoelectric material S_{te} , C_{te} , and Re_{Co} , Re_{He} , which include electrical resistances between the pieces of thermoelectric material and the characteristics of the heat exchangers, i.e., $RtBa_{Co}$, $RtBa_{He}$, $RtHy_{Co}$, $RtHy_{He}$, C_{xt} .

The equations are linear and the system is readily solved. The characteristics of the thermoelectric material are a function of their average temperature so an iteration method is necessary.

55.6 Inlet and Exit Equations

The above equations correspond to the thermoelectric building block, but as there are a succession of building blocks, the exit conditions from the inlet conditions² and the powers of each building block are calculated for each of the thermoelectric building blocks.

For noncondensing air and for a liquid such as water the following equations can be written: $tFL_{Co,ex} = tFL_{Co,in} + P_{Co}/Q_{Co} \cdot Cp_{Co}$ where Q_{Co} is the mass flow rate of the fluid (kg/s) and $Cp_{Co,in}$ is the specific heat of the fluid in J/(kg · K).

A model has been developed by Buffet and Stockholm^{2,3} for condensing air.

55.7 Calculations of a Unit

Fluids and Temperature

The level of complexity of the calculation depends on whether a gas (air) or a liquid (water) is being considered. A thermoelectric building block varies in size and depends on the amount of thermoelectric material per building block, that is, the total area of thermoelectric material at the cold junction. The area of thermoelectric elements ranges from 15 mm² to more than 150 mm² while the module can exceed 500 mm². The cooling is generally between 2 and 10 W per cm². Assuming values of 150 mm² and 3 W/cm², with a coefficient of performance (COP) of 1, this corresponds to a cooling power of 4.5 W/cm² and a heating power per building block of 9 W/cm².

The mass flow rate of water through the base of a building block containing tubes is of the order of 0.15 kg/s while in the case of air the mass flow rate is of the order of 10 g/s. With a cooling power of 100 W the corresponding changes in temperature ΔT between inlet and outlet of a unit for air and water are as follows:

$$\begin{aligned} 100 \text{ W} &= (0.010 \text{ kg/s}) \cdot (1006 \text{ J/(kg} \cdot \text{K)}) \cdot \Delta T \quad \text{so } \Delta T = 10\text{K} \\ 100 \text{ W} &= (0.15 \text{ kg/s}) \cdot (4,186 \text{ J/kg} \cdot \text{K}) \cdot \Delta T \quad \text{so } \Delta T = 0.16\text{K} \end{aligned}$$

Water-Water

Therefore, with a water-water unit containing 500 building blocks in series on the cooled water

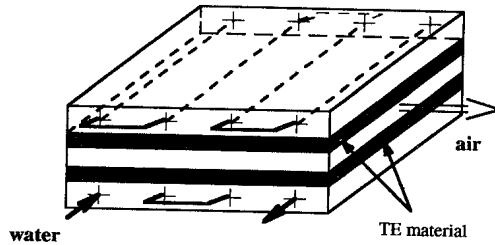


FIGURE 3 Water-air unit cross flow.

important. Buffet⁶ has shown that the optimization is quite different to that of passive heat exchangers with two liquids.

When the temperature variations of the water are 10°C or less it is sufficient to calculate the performances of the average building block. This is the building block with fluid temperatures that correspond to the average fluid temperature of each circuit. This means that it is only necessary to calculate one building block, the total power being equal to the power of the building block multiplied by their number.

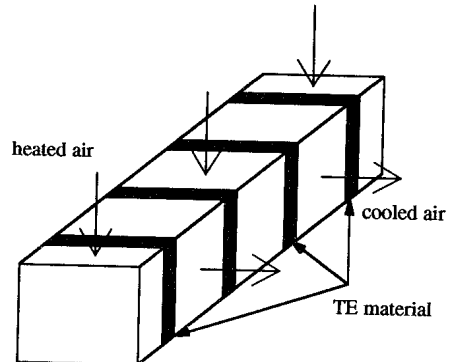
Water-Air

Consider the case of a water-air system containing, for example, 200 building blocks of which 50 are in parallel on the air and therefore 4 are in series. A schematic is shown in Figure 3; the individual building blocks are not shown. All the building blocks are in series in the water circuit, and as the water tube goes through the unit four times, there are four building blocks in series in the air circuit. The darkened areas correspond to the thermoelectric material.

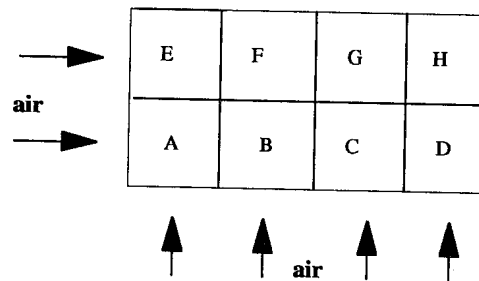
The calculation is carried out for each row of building blocks in parallel in the air circuit. The building block is calculated with a water temperature equal to the average temperature of the water in each row of building blocks. So only the performances of four building blocks are required.

Air-Air

An air-air unit is shown schematically in three dimensions in Figure 4. For this cross flow it is necessary to calculate each building block shown in Figure 5. The calculation sequence can be ABCD then EFGH or AE then BF, CG and DH. It does not make any difference which sequence is chosen in both cases all the building blocks are calculated. The total cooling power is obtained by adding the powers of each of the building blocks. The same is done for the heating and the electrical powers.



The average exit conditions are obtained by first



There are two possible ways of calculating the unit:

AE then BF then CG then DH

or ABCD then EFGH

FIGURE 5 Air-air unit cross-flow calculation.

55.8 Conclusions

The modeling of thermoelectric systems based on the concept of building blocks has been presented. The parameters that are necessary to characterize a building block are given, together with the procedure to calculate a system. It has been shown that for water-water systems it is only necessary to calculate the thermoelectric building block that "sees" the average hot-side and the average cold-side fluid temperatures. For water-air units the average thermoelectric building block in each row along the air circuit has to be calculated. For air-to-air systems each thermoelectric building block which has different inlet conditions has to be calculated.

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Notations

Symbol	Units	Designation
ABa_{Co}	m^2	Area of cooled base
ABa_{He}	m^2	Area of heated base
A_{Fin}	m^2	Area of fin
A_{Te}		Area of one thermoelectric element
C_{xt}	W/K	Thermal conductance of seal
C_{pCo}	J/(kg · K)	Heat capacity of cooled fluid
eff_{Fin}	dimensionless	Fin efficiency = average t of fin/t at base of fin
GF	m	Geometric factor of thermoelectric element
h_{Fin}	W/($m^2 \cdot K$)	Convection coefficient of fin
i	A	Electrical current
L_{Te}	m	Length of thermoelectric element
Nb_{Te}	dimensionless	Number of thermoelectric elements in the module
P_{Co}	W	Cooling power
P_{He}	W	Heating power
Q_{Co}	kg/s	Mass flow rate of cooled fluid
R_{Co}	Ω	Electrical resistance of cold-side heat exchanger if in electrical circuit
Re_{Co}	Ω	Electrical resistance $Re_{TE}/2 + R_{Co}$
Re_{He}	Ω	Electrical resistance $Re_{TE}/2 + R_{He}$
R_{He}	Ω	Electrical resistance of cold-side heat exchanger if in electrical circuit
$RtBa_{Co}$	K/W	Thermal resistance of cooled base
$RtBa_{He}$	K/W	Thermal resistance of heated base
$RtHy_{Co}$	K/W	Thermal hydraulic resistance of cooled base
$RtHy_{He}$	K/W	Thermal hydraulic resistance of heated base
t_{av}	$^{\circ}C$	Average temperature of thermoelectric material
tFL_{Co}	$^{\circ}C$	Temperature of cooled fluid
$tFL_{Co,ex}$	$^{\circ}C$	Temperature of cooled fluid at exit of building block
$tFL_{Co,in}$	$^{\circ}C$	Temperature of cooled fluid at inlet of building block
tFL_{He}	$^{\circ}C$	Temperature of heated fluid
tTe_{Co}	$^{\circ}C$	Temperature of thermoelectric material at the cooled end
tTe_{He}	$^{\circ}C$	Temperature of thermoelectric material at the heated end
ρ_{Te}	$\Omega \cdot m$	Thermoelectric material electrical resistivity
λ_{Te}	W/($m \cdot K$)	Thermoelectric material thermal conductivity
α_{Te}	V/K	Thermoelectric material Seebeck coefficient
Ct_{Te}	W/K	Thermoelectric thermal conductance = $Nb_{Te} \cdot$

Appendices

1. Convection coefficient of water in tubes

Generally considered, water flowing in circular ducts with a Reynolds number N_{Rey} in excess of 5000 with straight tubes having a length of less than 35 diameters. Experimentation has confirmed the formula given by MacAdams:

$$h = 1480 \cdot (1 + 0.015t)(V^{0.8})/D^{0.2} \text{ in SI units}$$

$$h = \text{convection coefficient } W/(m^2 \cdot K)$$

$$t = \text{temperature of the water } ^\circ C$$

$$V = \text{velocity of the water, m/s}$$

$$D = \text{diameter of the tube, m}$$

2. Convection coefficient of dry air

There exists many shapes of fins: flat fins, wavy fins, lanced fins, etc. Kays and London⁵ give the convection coefficient in a nondimensional form.

When dealing with air the convection coefficient h ($W/(m^2 \cdot K)$) can be expressed as $h = a \cdot V^b$ where a and b are constants for a specific heat exchanger and for a defined range of velocities V in m/s.

A typical formula for wavy fins in air around the ambient temperature with velocities between 2 and 10 m/s in turbulent flow is $h = 21 \cdot V^{0.635}$.